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Quark mixing, CKM unitarity

The unitarity problem

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Abstract. In the Standard Model of elementary particles, quark-mixing is expressed in terms of a 3 × 3 unitary matrix V, the so called Cabibbo-Kobayashi-Maskawa (CKM) matrix. Significant unitarity checks are so far possible for the first row of this matrix. This article reviews the experimental and theoretical information on these matrix elements. On the experimental side, we find a 2.2 σ to 2.7 σ deviation from unitarity, which conflicts with the Standard Model.

1 Introduction

The fundamental constituents of matter are the quarks and the leptons. The quark-mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix parametrizes the weak charged current interactions of quarks. The Standard Model does not predict the content of the CKM matrix; and the values of individual matrix elements are determined from weak decays of the relevant quarks. In this context, weak decays of nuclei, hadrons and CP violating processes play an important role. The CKM matrix is required to be unitary. One possible direct precision test of unitarity involves the top row of V, namely

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta. \tag{1}$$

In the Standard Model with a unitary CKM matrix, Δ is zero. The test fails for unknown reasons and a deviation from unitarity has been found with nuclear β-decay [1] and neutron-β-decay data [2].

Four parameters describe a unitary 3 × 3 matrix. The "standard" parametrization utilizes 3 angles and one phase. Table 1 shows the matrix elements of the CKM matrix as assessed by the Particle Data Group [3]. The range corresponds to 90% CL limits on the angles and the phase. The unitarity constraint has pushed $|V_{ud}|$ about two to three standard deviations higher than given by the experiments.

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Table 1. CKM quark-mixing matrix with 90%. C.L. from a global fit to angles and phase [3]. The unitarity constraint has pushed $|V_{ud}|$ about two to three standard deviations higher than given by the experiments

0.9741 to 0.9756	0.219 to 0.226	0.0025 to 0.0048
0.219 to 0.226	0.9732 to 0.9748	0.038 to 0.044
0.004 to 0.014	0.037 to 0.044	0.9990 to 0.9993

This article summarizes current knowledge on the CKM matrix from a workshop at Heidelberg in September 2002 [4]. The workshop reviewed the information to date on the inputs for the unitarity check from the experimental and theoretical side. Dedicated to quark-mixing of the first row of the CKM matrix, the meeting collected complementary information to the CKM workshops held at CERN and Durham [5,6] and had its main emphasis on $|V_{ud}|$. Due to its large size, a determination of $|V_{ud}|$ is most important. It has been derived from a series of experiments on superallowed nuclear β-decay through determination of phase space and measurements of partial lifetimes as will be explained in Sect. 2. With the inclusion of nuclear structure effect corrections, a value of $|V_{ud}| = 0.9740(5)$ [1] emerges in good agreement of different, independent measurements in nine nuclei. Combined with $|V_{us}| = 0.2196(23)$ from kaon-decays and $|V_{ub}| = 0.0036(9)$ from B-decays, this leads to $\Delta = 0.0031(14)$, signaling a deviation from the Unitarity condition by 2.2 σ standard deviations. The quoted uncertainty in $|V_{ud}|$ is dominated by the uncertainties in the theoretical correction terms, and, as described in Sect. 2, current nuclear experiments are focused on testing and refining those correction terms that depend on nuclear structure. Such terms are avoided entirely in neutron β-decay (see Sect. 3) and in pion β-decay. Recently, the mixing of the down quark has been studied in the decay of free neutrons. With the measurement of the neutron decay β-asymmetry A_0, using a highly polarized cold neutron beam with an improved instrument and the world average of the neutron lifetime τ one is now capable of extracting a value for the first entry of the CKM-quark-mixing matrix, whilst avoiding large corrections to the raw-data or problems linked to nuclear structure. With neutron-decay data, $|V_{ud}| = 0.9717(13)$ leads to significant deviation $\Delta = 0.0076(28)$, which is 2.7 times the stated error. The pion β-decay has been measured recently at the PSI. The pion has a different hadron structure compared with neutron or nucleons and it offers another possibility in determining $|V_{ud}|$. The preliminary result is $|V_{ud}|$=0.9771(56) [7]. The error is still too large to allow a significant unitarity check.

A violation of unitarity in the first row of the CKM matrix is a challenge to the three generation Standard Model. The data available so far do not preclude there being more than three generations; CKM matrix entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations [3,8]. A deviation Δ has been related to concepts beyond the Standard Model, such as couplings to exotic fermions [9, 10], to the existence of an additional Z boson [11, 12], to supersymmetry or to the existence of right-handed currents in the weak interaction [13]. Non-unitarity of the CKM matrix in models with an extended quark sector give rise to an induced neutron electric dipole moment that can be within reach of the next generation of experiments [14].

2 $|V_{ud}|$ from superallowed $0^+ \to 0^+$ beta decay

Currently, superallowed $0^+ \to 0^+$ nuclear β-decay provides the most precisely determined value for V_{ud}, the up-down quark mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This value is also the most precise result for any element in the CKM matrix and leads to the most demanding test available of CKM unitarity, a test which apparently fails by more than two standard deviations [15, 16]: With $|V_{us}| = 0.2196(23)$ and the negligibly small $|V_{ub}|$ = 0.0036(9), one gets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta = 0.9969 \pm 0.0014, \quad (2)$$

and $\Delta = 0.0031(14)$. Nuclei have the singular advantage that transitions with specific desirable characteristics can be selected and then isolated for study. The case of $0^+ \to 0^+$ β-transitions between $T = 1$ analog states is an excellent example, since angular momentum limits such transitions uniquely to the vector part of the weak interaction, thus eliminating the need for special experiments designed to account separately for the vector and axial-vector parts. Furthermore, the nuclear matrix element is given by the expectation value of the isospin ladder operator and, since the parent and daughter states for these transitions are analogs of one another, the result should be independent of any details of nuclear structure to the extent that isospin symmetry is preserved. Of course, there are corrections to this simple picture, originating from charge-dependent mixing and other electromagnetic effects, but these corrections are small – of order 1% – and calculable.

The measured intensity of a particular β-transition is expressed as an ft-value. This ft-value is determined by three measured parameters: the transition energy Q_{EC}, which is used in calculating the statistical rate function, f; the half-life of the β-emitter and the branching ratio for the transition of interest, which together yield the partial half-life, t. The experimentally determined ft-value relates to the Fermi constant, G_F via the relationship [15, 16]

$$\mathcal{F}t \equiv ft\,(1 + \delta'_R + \delta_{NS})\,(1 - \delta_C) = \frac{K}{2|V_{ud}|^2 G_F^2\,(1 + \Delta_{\mathrm{R}}^{\mathrm{V}})}, \quad (3)$$

where K is a known constant. The small correction terms comprise δ_C, the isospin-symmetry-breaking correction; δ'_R and δ_{NS}, the transition-dependent parts of the radiative correction; and $\Delta_{\mathrm{R}}^{\mathrm{V}}$, the transition-independent part. Here we have also defined $\mathcal{F}t$ as the "corrected" ft-value. Note that, of the four calculated correction terms, two – δ_C and δ_{NS} – depend on nuclear structure and their influence in (3) is effectively in the form $(\delta_C - \delta_{NS})$.

To date, there are nine nuclear $0^+ \to 0^+$ transitions whose ft-values have been measured with a precision of $\sim 0.1\%$ or better. Many separate measurements and experimental teams have contributed to this body of data and the results can be considered extremely robust, most input data having been obtained from several independent and consistent measurements [15, 17]. The decay parents – ^{10}C, ^{14}O, ^{26m}Al, ^{34}Cl, ^{38m}K, ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co – also span a wide range of nuclear masses. Nevertheless, as anticipated by the Conserved Vector Current hypothesis, CVC, all nine yield consistent $\mathcal{F}t$-values and hence a unique value for G_V.

The value of V_{ud} is obtained by relating the vector constant, G_V, determined from the self-consistent nuclear $\mathcal{F}t$-values, to the weak coupling constant from muon decay. This result, $V_{ud} = 0.9740 \pm 0.0005$, leads to the unitarity test already displayed in (2). (In deriving these results we have used the Particle Data Group's [3] recommended values for the muon coupling constant and for V_{us} and V_{ub}.) It is informative to dissect the contributions to the uncertainty obtained for V_{ud}. The contributions to the overall ± 0.0005 uncertainty are 0.0001 from experiment, 0.0001 from δ'_R, 0.0003 from $(\delta_C - \delta_{NS})$, and 0.0004 from $\Delta_{\mathrm{R}}^{\mathrm{V}}$. Thus, if the unitarity test is to be sharpened, then the most pressing objective must be to reduce the uncertainties on $\Delta_{\mathrm{R}}^{\mathrm{V}}$ and $(\delta_C - \delta_{NS})$.

Improvements in $\Delta_{\mathrm{R}}^{\mathrm{V}}$ are a purely theoretical challenge, the solution of which will not depend on further experiments. However, experiments can play a role in improving the next most important contributor to the uncertainty on V_{ud}, namely $(\delta_C - \delta_{NS})$. Clearly this correction applies only to the results from superallowed beta decay and, in the event that improvements are made in $\Delta_{\mathrm{R}}^{\mathrm{V}}$, will then limit the precision with which V_{ud} can be determined by this route. Recently, a new set of consistent calculations for $(\delta_C - \delta_{NS})$ have appeared [16] not only for the nine well known superallowed transitions but for eleven other superallowed transitions that are potentially accessible to precise measurements in the future. Experimental activity is now focused on probing these nuclear-structure-dependent corrections with a view to reducing the uncertainty that they introduce into the unitarity test.

2.1 The future

The approach being taken by current experiments is to choose as yet unmeasured superallowed transitions for which it is predicted that the structure-dependent corrections, $(\delta_C - \delta_{NS})$, are particularly large, or to choose several such transitions that cover a wide range of calculated corrections. If a transition with a much larger correction than any currently known yields an $\mathcal{F}t$-value that also agrees with the current average – i.e., is consistent with CVC – then this would constitute a critical test of the accuracy of the calculated structure-dependent corrections. If such measurements are found to support the calculations, then this would validate those calculations and act to reduce the uncertainties attributed to them, uncertainties that currently are based only on theoretical estimates.

Experimental attention is currently focused on two series of 0^+ nuclei: the even-Z, $T_z = -1$ nuclei with $18 \leq A \leq 42$, and the odd-Z, $T_z = 0$ nuclei with $A \geq 62$. Both regions include transitions with larger calculated values [16] for $(\delta_C - \delta_{NS})$ than any of the nine currently well-known transitions. Of the heavier $T_z = 0$ nuclei, ^{62}Ga and ^{74}Rb are receiving the greatest attention at this time (see [18] and experimental references therein). The decays of nuclei in this series are of higher energy than any previously studied and each therefore involves numerous weak Gamow-Teller transitions in addition to the superallowed transition [18]. Branching-ratio measurements are thus very demanding, particularly with the limited intensities likely to be available initially for most of the rather exotic nuclei in this series. Nevertheless, with the help of shell-model calculations [18], a combination of detailed β- and γ-ray spectroscopic measurements has recently been used to obtain a precise value for the superallowed branching ratio in the decay of prolifically produced ^{74}Rb [19].

More accessible in the short term are the $T_z = -1$ superallowed emitters with $18 \leq A \leq 42$. The nuclear-model space used in the calculation of $(\delta_C - \delta_{NS})$ for these nuclei is exactly the same as that used for some of the nine transitions already studied. If the wide range of values predicted for the corrections are confirmed by the measured ft-values, then it will do much to increase our confidence (and reduce the uncertainties) in the corrections already being used. To be sure, these decays also provide an experimental challenge, particularly in the measurement of their branching ratios, which involve *strong* Gamow-Teller transitions, but sufficiently precise results have just been obtained [20] for the half life and superallowed branching ratio for the decay of ^{22}Mg and work on ^{34}Ar decay is well advanced. New precise ft-values should not be long in appearing. It would be virtually impossible for them to have any effect on the central value already obtained for V_{ud} but they may be expected ultimately to lead to reduced uncertainties on that value.

3 $|V_{ud}|$ from neutron β-decay

Recently, $|V_{ud}|$ has been derived not from nuclear β-decay but from neutron decay data [2]. In this way, the unitarity check of (1) is based solely on particle data, i.e. neutron β-decay, K-decays, and B-decays, where nuclear structure effects are absent. So much progress has been made using highly polarized cold neutron beams with an improved detector setup that one is now capable of extracting a value $|V_{ud}| = 0.9717(13)$. Here, the unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta = 0.9924(28) \qquad (4)$$

fails by $\Delta = 0.0076 \pm 0.0028$, or 2.7 times the stated error σ. Earlier experiments [21–23] gave significantly lower values for the neutron decay β-symmetry A_0. Averaging over the new result and previous results, the Particle Data Group [3] arrives at a new world average for $|V_{ud}|$ from neutron β-decay which leads to a 2.2 σ deviation from unitarity.

Since the Fermi decay constant is known from muon decay, the Standard Model describes neutron β-decay with only two additional parameters. One parameter is the first entry $|V_{ud}|$ of the CKM-matrix. The other one is λ, the ratio of the vector coupling constant and the axial vector constant. In principle, the ratio λ can be determined from QCD lattice gauge theory calculation, but the results of the best calculations vary by up to 30%. In neutron decay, several observables are accessible to experiments, which depend on these parameters, so the problem is overdetermined and, together with other data from particle and nuclear physics, many tests of the Standard Model become possible. $|V_{ud}|$ results significantly from the neutron lifetime τ and the β-asymmetry parameter A_0. The ft-value is given by

$$ft(1 + \delta'_R) = \frac{K}{|V_{ud}|^2 G_F^2 \,(1 + 3\lambda^2)\,(1 + \Delta_R)}, \qquad (5)$$

where f =1.6886 is the phase space factor. The model independent radiative correction δ'_R and other small terms changes the phase space factor by 1.5% to $f^R = 1.71335(15)$ [24, 25]. $\Delta_R = 0.0240(8)$ is the model dependent radiative correction to the neutron decay rate [1, 26]. The β-asymmetry A_0 is a simple function of λ, the ratio of the axial vector to vector coupling constant

$$A_0 = -2\frac{\lambda(\lambda + 1)}{1 + 3\lambda^2}, \qquad (6)$$

where we have assumed that λ is real.

With recent experiments [2, 27], one obtains $A_0 = -0.1189(7)$ and λ = -1.2739(19). With this value, and the world average for $\tau = 885.7(7)$ s, one finds that $|V_{ud}| =$

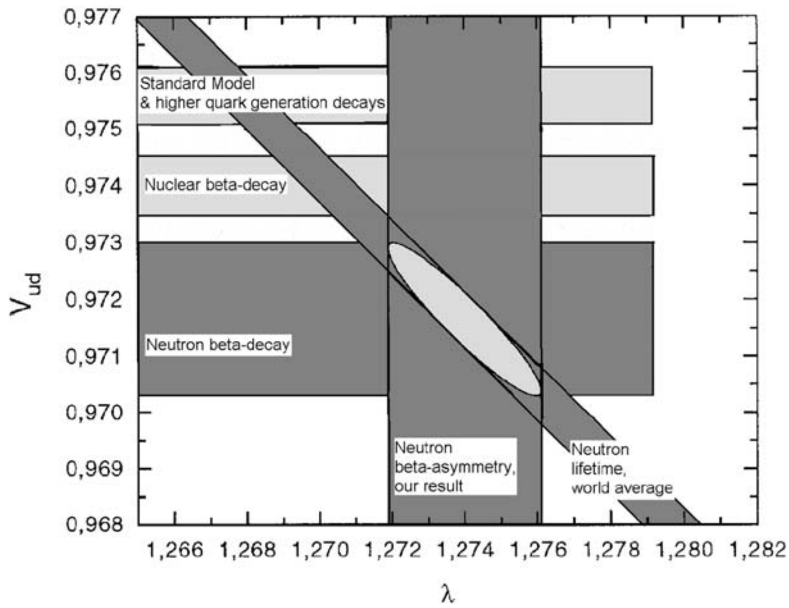

Fig. 1. $|V_{ud}|$ vs. λ. $|V_{ud}|$ was derived from higher quark generation decays via $|V_{ud}| = \sqrt{1 - |V_{us}|^2 + |V_{us}|^2}$ predicted from unitarity, from Ft values of nuclear β-decays, and neutron β-decay

0.9717(13). The main contribution to the overall ± 0.0013 uncertainty is the experimental error from the β-asymmetry A_0 with ± 0.0012. In these β-asymmetry experiments, the total correction to the raw data is 2.0%. We favour this result over earlier experiments [21–23], where large corrections had to be made for neutron polarization, electron-magnetic mirror effects or background, which were all in the 15% to 30% range. The world average value for the neutron mean lifetime includes 11 individual measurements, using different techniques summarized as "beam methods" and "bottle methods". The measurements agree nicely with a χ^2 of 0.95 but the world average value is dominated by a single experiment [28]. The error contribution to $|V_{ud}|$ with ± 0.0004 from the neutron lifetime is small and the same as the error from Δ_R. The contribution to f from δ'_R is with ± 0.00004 completely negligible.

For comparison, information about $|V_{ud}|$ and λ are shown in Fig. 1. The bands represent the one sigma error of the measurements. The β-asymmetry A_0 in neutron decay depends only on λ, while the neutron lifetime τ depends both on λ and $|V_{ud}|$. The intersection between the bands derived from τ and A_0 defines $|V_{ud}|$ within one standard deviation, which is indicated by the error ellipse. For comparison, $|V_{ud}|$ derived from nuclear β-decay as well as a value $|V_{ud}| = \sqrt{1 - |V_{us}|^2 + |V_{us}|^2}$ predicted from unitarity are shown, too.

3.1 The future

The main corrections in recent neutron decay asymmetry experiments [2] are due to neutron beam polarization (1.1%), background (0.5%) and flipper efficiency (0.3%). The total correction is 2.0%, the total relative error is 0.68%. For the future, the plan is to further reduce all corrections. Major improvements both in neutron flux and degree of neutron polarization have already been made: First, the new ballistic supermirror guide at the Institute-Laue Langevin in Grenoble gives an increase of about a factor of 4 in the cold neutron flux [29]. Second, a new arrangement of two supermirror polarizers allows to achieve an unprecedented degree of neutron polarization P of between 99.5% and 100% over the full cross section of the beam [30]. Third, systematic limitations of polarization measurements have been investigated: The beam polarization can now be measured with a new method using an opaque ^{3}He spin filter with an uncertainty of 0.1% [31,32]. As a consequence, we will be able to improve on the main uncertainties in reducing the main correction of 1.1% to less than 0.5% with an error of 0.1%. Future trends have been presented on the workshop "Quark-mixing, CKM Unitarity" [4]. Regarding lifetime measurements, several independent experiments with a projected accuracy of one second or better are carried out. One of them uses the storage of ultra-cold neutrons in a material trap with a gravitational valve [33]. The coating of the trap surface allows to obtain a storage time of 870s, which is very close to the neutron lifetime. The use of two traps with different sizes and the method of size extrapolation expect an accuracy for the neutron lifetime at the level of one second. In a different approach, neutrons are trapped magnetically with permanent magnets [34] or with the superconducting magnets [35,36]. Regarding the unitarity problem, about half a dozen new instruments are planned or are under construction for beta-neutrino correlation a and beta-correlation A measurements at the sub-10^{-3} level, which should result in a value of $|V_{ud}|$ whose error is dominated by the theoretical uncertainties in the radiative corrections (see Sect. 4).

4 Electroweak radiative corrections

In order to obtain accurate values for the V_{ud} element of the CKM matrix from nuclear, neutron and pion beta decays, we need precise radiative correction calculations for their observables. For this purpose, one has to deal with several different kinds of Feynman diagrams. In addition to the W boson mediating the beta decay process, further electroweak bosons (photon, W and Z bosons) can be created and absorbed by the fermions, and these bosons can change slightly the various decay probabilities.

In order to compute the radiative corrections, both the virtual and the photon bremsstrahlung contributions have to be evaluated. Usually, the bremsstrahlung photons are not detected in beta decay experiments, therefore one has to integrate these photons down to zero energy. These bremsstrahlung integrals are infinite: they have infrared divergency. However, adding the contributions of the virtual diagrams to the bremsstrahlung integrals, the infrared divergency disappears. The bremsstrahlung photons can leave the small space-time region of the beta decay, and their momenta can be of similar magnitude as the momenta of the other particles involved in the decay process. Therefore, these photons can change the beta decay kinematics (in contrast to the virtual photons, which have no effect to the kinematics). It is important to take into account this fact, in order to obtain meaningful radiative correction results. We refer to [37,38] for detailed explanation of this photon bremsstrahlung kinematics effect. The photon bremsstrahlung calculation is theoretically simple and reliable. Due to their small energy, the bremsstrahlung photons see only the hadron charges, therefore this part of the radiative correction calculation has no uncertainties related with strong interaction models. On the other hand, technically it is more complicated. In order to evaluate the many-dimensional integrals occurring in the computation of arbitrary observables, the Monte Carlo method seems to be expedient to use [39].

In order to compute the virtual correction, the energy and momentum of the virtual photon has to be integrated from zero to infinity. The small energy part of the virtual integrals has similar properties to the bremsstrahlung correction (infrared divergence, almost no strong interaction dependence, sensitivity to external particle momenta).
Therefore, it is expedient to separate this part of the virtual correction from the larger photon energy region contributions. Sirlin has introduced in his prominent paper [40] a separation of the order-α radiative correction into model independent (MI) and model dependent (MD) parts. The MI (outer) correction is defined as the sum of the photon bremsstrahlung correction and the small energy part of the virtual correction. The MD (inner) correction contains the medium and large energy (asymptotic) parts of the virtual contributions. It was proved in [40] that neglecting some small terms of order 0.01 %, the MD correction can be absorbed into the dominant vector and axial vector form factors and is not taken into account in a determination of λ from the β-asymmetry parameter A_0.

The order-α MI radiative corrections to the total decay rates of allowed beta decays can be simply computed by the universal Sirlin function [40]. For example, in neutron decay this correction increases the decay rate by 1.5 %. On the other hand, the MI correction to the asymmetry parameters is rather small [38,42–44]. Due to this correction, the value of the parameter λ from a measurement of the β-asymmetry parameter is changed by 0.0003, which is far below the present experimental error of 0.0019. Other observables for determining λ have larger corrections. For example for the proton spectrum in unpolarized neutron decay, the change of λ is 0.01 due to the MI correction [45] and the main part of this large correction comes from the photon bremsstrahlung kinematic effect mentioned above.

4.1 One and two loop electroweak corrections

Modulo the Fermi function, electroweak radiative corrections to superallowed $0^+ \to 0^+$ nuclear beta decays are traditionally factored into two contributions called inner and outer corrections. The outer (or long distance) correction $1 + \delta'_R + \delta_{NS}$ is given by

$$1 + \frac{\alpha}{2\pi}\left(g\left(E, E_{\max}\right) + 2C_{NS}\right) + \delta_2\left(Z, E\right), \qquad (7)$$

where $g(E, E_{\max})$ is the universal Sirlin function [40] which depends on the nucleus through $E_{\max}$, the positron or electron end point energy. C_{NS} is a nuclear structure dependent contribution induced by axial-current nucleon-nucleon interactions [46] and δ_2 is an $O(Z\alpha^2)$ correction partly induced by factorization of the Fermi function and outer radiative corrections [47].

The contribution from $g(E, E_{\max})$ is quite large ($\sim$ 1.3% $for\ O^{14}$) due to a $3\ln(m_p/E_{\max})$ term which generally dominates. Summation of $(\alpha \ln m_p/E_{\max})^n$, $n = 2, 3\ldots$ contributions from higher orders gives an additional 0.028% correction [25] while additional $O(\alpha^2)$ effects are estimated to be $< 0.01\%$.

C_{NS} and $\delta_2(Z, E)$ are nucleus dependent. The leading contribution to δ_2 is of the form $Z\alpha^2 \ln m_p/E$ where Z is the charge of the daughter nucleus. Just as in the case of the Fermi function, δ_2 is usually given for positron emitters (since that is appropriate for superallowed decays). For electron emitters the sign of Z should be changed in both the Fermi function and $\delta_2(Z, E)$. Unfortunately, as pointed out by Czarnecki, Marciano and Sirlin [25], that sign change was not made in the case of neutron decay. As a result, the often quoted 0.0004 contribution from δ_2 to neutron β-decay should be changed to -0.00043, an overall shift of -0.083%. With those corrections, the overall uncertainty in the outer radiative corrections is now estimated to be about $\pm 0.01\%$.

The inner radiative correction factor $1 + \Delta_{\mathrm{R}}^{\mathrm{V}}$ is given (at one loop level) by

$$1 + \frac{\alpha}{2\pi}\left(4\ln\frac{m_Z}{m_p} + \ln\frac{m_p}{m_A} + A_g + 2C\right), \qquad (8)$$

where the $\frac{2\alpha}{\pi}\ln m_Z/m_p \simeq 0.0213$ universal short- distance correction dominates [48]. The contributions induced by axial- vector effects are relatively small but carry the bulk of the theoretical uncertainty

$$\frac{\alpha}{2\pi}\left[\ln\frac{m_p}{m_A} + A_g + 2C\right] \simeq -0.0015 \pm 0.0008. \qquad (9)$$

It stems from an uncertainty in the effective value of m_A that should be employed. The quoted uncertainty in (3) allows for a conservative factor of 2 uncertainty in that quantity. It would be difficult to significantly reduce the uncertainty for nuclei or the neutron. In the case of pion beta decay, the uncertainty is likely to a factor of 2 or more smaller.

High order $(\alpha \ln m_Z/m_p)^n, n = 2, 3 \ldots$ leading log contributions are expected to dominate the multi-loop effects. They have been summed by renormalization group techniques [8], resulting in an increase in (8) by 0.0012. Next to leading logs of $O(\alpha^2 \ln m_Z/m_p)$ have been estimated to give -0.0002 ± 0.0002 while $O(\alpha^2)$ effects are expected to be negligible. In total, a recent update finds [25]

$$\text{Inner R. C. Factor} = 1.0240 \pm 0.0008, \qquad (10)$$

which is essentially the same as the value given by Sirlin in 1994 [49]. It leads to

$$|V_{ud}| = 0.9740 \pm 0.0001 \pm 0.0001 \pm 0.0003 \pm 0.0004, \quad (11)$$

extracted from super-allowed beta decays, where the errors stem from the experimental uncertainty, the two transition dependent parts of the radiative corrections δ'_R and $\delta_C - \delta_{NS}$, and the inner radiative correction Δ_R^V respectively.

In the case of neutron decay, the radiative corrections carry a similar structure and uncertainty. Correcting for the sign error in the $Z\alpha^2$ effect, one finds the master formula [25]

$$|V_{ud}|^2 = \frac{4908 \pm 4sec}{\tau_n\,(1+3\lambda^2)}. \qquad (12)$$

Employing $\tau_n = 885.7(7)$s and $\lambda = 1.2739(19)$ then implies

$$|V_{ud}| = 0.9717 \pm 0.0004 \pm 0.0012 \pm 0.00004 \pm 0.0004 \quad (13)$$

where the errors stem from the experimental uncertainty in the neutron lifetime, the β-asymmetry A_0 and the theoretical outer and inner radiative correction δ'_R and Δ_R^V respectively. In the case of pion beta decay, the theory uncertainty in $|V_{ud}|$ is probably ± 0.0002 or smaller, but the small ($\simeq 10^{-8}$) branching ratio makes a precision measurement very difficult.

5 $|V_{us}|$ from hyperon and kaon-decays

Hyperon or K decays determine V_{us}. The analysis of hyperon data has larger theoretical uncertainties because different calculations of SU(3) symmetry breaking effects disagree. Therefore the Particle Data Group relies on the K decay data, based on a derivation in 1984 [50].

The rates of K_{l3} has the form [50, 51]

$$\Gamma = \frac{G_F^2}{192\pi^3} M_k^5 |V_{us}|^2 C^2 |f(0)|^2 |I(1+\delta)(1+\Delta) \qquad (14)$$

with phase space integral I, form factor f, M_k the kaon mass, Clebsch-Gordan coefficient C^2 and radiative corrections [52, 53] $\Delta = 2.12 \pm 0.08\%$ and δ = -2% for K_{e3}^+ decays and $\delta = 0.5\%$ for K_{e3}^0 decays. Updates of $|V_{us}|$ with revised radiative corrections [54] are in agreement with the current PDG value $|V_{us}| = 0.2196 \pm 0.0023$ [3, 50] and even indicate a decrease of the central value by up to 1%. However, a very recent report of the K_{e3}^+ branching ratio from E865 at BNL results in a larger $|V_{us}| = 0.2272(30)$ [55]. With this value of $|V_{us}|$ alone, we find no significant deviation from CKM unitarity. On the other hand, the discrepancy between this BNL $|V_{us}|$ value and a value from K_{e3}^0 is on the 3 σ level. New, ongoing or prepared K_{e3} measurements (e.g. CMD2, NA48, KTEV, KLOE) [4, 6] will help to solve this K-decay problem. After this workshop, a new analysis of hyperon decays by Cabibbo, Swallow and Winston that is insensitive to first order SU(3) breaking effect appeared [56]. It found $|V_{us}| = 0.2250(27)$ which is in better agreement with unitarity and the E865 K_{e3} results than previous studies, thus providing additional motivation for further experimental work on $|V_{us}|$.

Table 2. Current and expected precision of CKM matrix elements. Prospective measurements are listed for linear e^+e^- colliders. The second column is taken from [59]. The second error is due to an uncertainty of Γ_{top} of 1%

	current uncertainty	projected for new collider
$\lvert V_{ud}\rvert$	± 0.0005	± 0.0028
$\lvert V_{us}\rvert$	± 0.0023	± 0.0124
$\lvert V_{ub}\rvert$	± 0.008	± 0.011
$\lvert V_{cd}\rvert$	± 0.016	± 0.0072
$\lvert V_{cs}\rvert$	± 0.16	± 0.0017
$\lvert V_{cb}\rvert$	± 0.0019	± 0.11
$\lvert V_{td}\rvert$	$\lvert V_{td}\rvert/\lvert V_{ts}\rvert < 0.24$	$\pm 0.026 \pm 0.35$
$\lvert V_{ts}\rvert$	± 0.008	$\pm 0.006 \pm 0.0002$
$\lvert V_{tb}\rvert$	$+0.29, -0.12)$	$\pm 0.000008 \pm 0.005$

6 CKM matrix elements from decays of W bosons and top quarks

Decays of $W^\pm$ bosons produced at LEP2 have been used to measure the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$ with a precision of 1.3% without the need of a form factor. The same data set has been used to test the unitarity of the first two rows of the matrix at the 2% level. At a future e^+e^- linear collider, with a data sample of few million of W decays a precision of 0.1% can be reached.

6.1 Determination of the W branching ratio

The observation of W decays offers another way to determine the CKM matrix elements. In the Standard Model the branching fraction of the W boson decays depends on the six CKM matrix elements which do not involve the top quark. Measuring the W production rates for different flavours gives access to the individual CKM matrix elements without parameterization of non-perturbative QCD:

$$\Gamma(\mathrm{W} \to \mathrm{q}'\bar{\mathrm{q}}) = \frac{\mathrm{C}(\alpha_\mathrm{s})\mathrm{G_F M_W^3}}{6 \cdot \sqrt{2}\pi} |\mathrm{V_{ij}}|^2 = (707 \pm 1)|\mathrm{V_{ij}}|^2 \mathrm{MeV},$$

where

$$C(\alpha_s) = 3 \left[1 + \sum_{i=1,3} \frac{a_i \alpha_s(\mathrm{M_W^2})}{\pi} \right]$$

is the QCD colour factor, up to the third order in $\alpha_s(\mathrm{M_W^2})$, the strong coupling constant. Furthermore, 'on shell' W bosons decay before the hadronization process starts, and the quark transition occurs in a perturbative QCD regime. Hence, W boson decays offer a complementary way to determine the CKM matrix elements.

In a similar way one can relate the top quark transition $B(t \to qW)$ to the CKM matrix elements $|V_{tq}|$.

From 1997 to 2000 the LEP e^+e^-collider has been operated at energies above the threshold for W-pair production. This offered a unique opportunity to study the hadronic decays of W boson in a clean environment and to investigate the coupling strength of W bosons to different quark flavours.

The leptonic branching fraction of the W boson $\mathcal{B}(\mathrm{W} \to \ell\overline{\nu}_\ell)$ is related to the six CKM elements not involving the top quark by:

$$\frac{1}{\mathcal{B}(\mathrm{W} \to \ell\overline{\nu}_\ell)} = 3 \left\{ 1 + \left[1 + \frac{\alpha_s(\mathrm{M_W^2})}{\pi} \right] \sum_{\substack{i\ =(u,c),\\ j\ =(d,s,b)}} |\mathrm{V_{ij}}|^2 \right\}.$$

Using $\alpha_s(\mathrm{M_W^2})$=0.119 $\pm$ 0.002, the measured leptonic branching fraction of the W yields

$$\sum_{\substack{i\ =(u,c),\\ j\ =(d,s,b)}} |\mathrm{V_{ij}}|^2 = 2.026 \pm 0.026 \pm 0.001, \qquad (15)$$

where the first error is due to the uncertainty on the branching fraction measurement and the second to the uncertainty on α_s [57]. This result is consistent with the unitarity of the first two rows of the CKM matrix at the 1.5% level, as in the Standard Model:

$$\sum_{\substack{i\ =(u,c),\\ j\ =(d,s,b)}} |\mathrm{V_{ij}}|^2 = 2. \qquad (16)$$

No assumption on the values of the single CKM elements are made.

Using the experimental knowledge [3] of the sum $|\mathrm{V_{ud}}|^2+|\mathrm{V_{us}}|^2+|\mathrm{V_{ub}}|^2+|\mathrm{V_{cd}}|^2+|\mathrm{V_{cb}}|^2 = 1.0477{\pm}0.0074$, the above result can also be interpreted as a measurement of $|\mathrm{V_{cs}}|$:

$$|\mathrm{V_{cs}}| = 0.989 \pm 0.014.$$

The error includes a $\pm$0.0006 contribution from the uncertainty on α_s and a $\pm$0.004 contribution from the uncertainties on the other CKM matrix elements, the largest of which is that on $|\mathrm{V_{cd}}|$. These contributions are negligible compared to the experimental error from the measurement of the W branching fractions, $\pm$0.014.

The W decay branching fractions, $\mathcal{B}(\mathrm{W} \to \mathrm{f}\bar{\mathrm{f}}')$, are determined from the cross sections for the individual WW→ 4f decay channels measured by the four experiments at all energies above 161 GeV. These branching fractions can be derived with and without the assumption of lepton universality. In the fit with lepton universality, the branching fraction to hadrons is determined from that to leptons by constraining the sum to unity.

Assuming lepton universality, the measured hadronic branching fraction is 67.77 $\pm$ 0.18(stat.) $\pm$ 0.22(syst.)% and the measured leptonic branching fraction is 10.74 $\pm$ 0.06(stat.)$\pm$0.07(syst.)%. These results are consistent with the Standard Model expectations, 67.51% and 10.83% respectively [58]. In this case, the high χ^2 of 20.8 for 11 degrees degrees of freedom is mainly due to the L3 results for W decays to muons and taus.

7 Conclusion

The presently poorly satisfied unitarity condition for the CKM matrix presents a puzzle in which a deviation Δ from unitarity may point towards new physics. The unitarity tests fails by up to 2.7 standard deviations. The origin of deviation Δ is unclear.

In $|V_{ud}|$ from nuclear $0^+ \to 0^+$ transitions with deviation Δ = 0.0032$\pm$0.0014, the error is dominated by theoretical uncertainties, where errors of nuclear structure dependent corrections are no larger than those of transition independent corrections. Restoration of unitarity would require a 2.3 σ shift of these corrections. In $|V_{ud}|$ from neutron decay, with deviation Δ = 0.0076$\pm$0.0028, the error is dominated by experimental uncertainties. Restoration of unitarity would require a 3 σ shift in the present value of the β-decay asymmetry A, or a 8 σ shift in the neutron lifetime τ. Alternatively, radiative corrections would have to be wrong by 8 σ. If the deviation is due to errors in $|V_{us}|$, its presently accepted value would have to shift by 7 σ in order to explain the neutron result, or by 3 σ to explain the nuclear $0^+ \to 0^+$ result. However, very recent preliminary results hint that the last world on $|V_{us}|$ is not yet spoken. On the other hand, $|V_{ub}|$ is completely negligible in this context. Other sources such as pion or W decay may have smaller theoretical errors, but their present experimental errors are not yet competitive.

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